ON A CONJECTURE CONCERNING EXACTLY COVERING SYSTEMS OF CONGRUENCES

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ABSTRACT

A conjecture of S. Znam [1] is disproved by means of a counterexample.

The concept of Covering System was introduced by P. Erdös [2]. A covering system is a set of ordered pairs of integers (a_i, n_i) $a_i \geqq 0$, $n_i > 1$ and

$$n_i \neq n_j \quad \text{for} \quad i \neq j \tag{1}$$

such that every integer satisfies *at least* one of the congruences $x \equiv a_i (\bmod n_i)$. An Exactly Covering System is defined [2] similarly but omitting condition (1) and requiring that every integer satisfies *exactly* one of the congruences $x \equiv a_i$ $(\bmod n_i)$.

Znam conjectured [1] that if in an exactly covering system there exist only pairs of equal moduli (no three being equal) then all moduli are of the form $n_i = 2^{\alpha_i} 3^{\beta_i}$, where α_i and β_i are non-negative integers.

We disprove this conjecture by giving the following counter-example:

$$1;\ 2 \pmod{5}$$

$$3;\ 4 \pmod{10}$$

$$5;\ 10 \pmod{15}$$

$$8;\ 18 \pmod{20}$$

$$9;\ 19 \pmod{30}$$

$$0;\ 15 \pmod{45}$$

$$29;\ 59 \pmod{60}$$

$$30;\ 75 \pmod{90}$$

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REFERENCES

1. S. Znam, On exactly Covering Systems of Arithmetic Sequences, *Math. Ann.* **180** (1969), 227–232.

2. P. Erdös, On a Problem Concerning Congruence Systems, *Math. Lapok* 3 (1952), 122–128.

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